Lecture Notes 4

1. Confidence Intervals

Point estimate is a single number, like $\hat{\beta}$

To calculate a confidence interval, we start with the following equation
:

$$\Pr\left(-t_c \leq \frac{\hat{\beta}_j - \beta_j}{std.error\left(\hat{\beta}_j\right)} \leq t_c\right) = 1-\alpha$$

where Pr denotes probability
t_c the critical value from the t-distribution
associated with a
α is the level of significance, usually $\alpha = 5\%$
$1 - \alpha$ is the chance that the true parameter lies within that interval
$\hat{\beta}$ and $std.error(\beta)$ are estimated values from your OLS regressions.
β is the true value.

$$\Pr\left(-t_c \cdot std,error\left(\hat{\beta}\right) \leq \hat{\beta}_j - \beta \leq t_c \cdot std,error\left(\hat{\beta}\right)\right) = 1-\alpha \; .$$

Subtracting $\hat{\beta}_j$ from both sides gives:

$$\Pr\left(-\hat{\beta}_j - t_c \cdot std,error\left(\hat{\beta}\right) \leq -\beta \leq -\hat{\beta}_j + t_c \cdot std,error\left(\hat{\beta}\right)\right) = 1-\alpha \; .$$

Multiplying by -1 gives:

$$\Pr\left(\hat{\beta}_j + t_c \cdot std,error\left(\hat{\beta}\right) \geq \beta \geq \hat{\beta}_j - t_c \cdot std,error\left(\hat{\beta}\right)\right) = 1-\alpha \; .$$

Rearranging terms gives:

(4) $$\Pr\left(\hat{\beta}_j - t_c \cdot std,error\left(\hat{\beta}\right) \leq \beta \leq \hat{\beta}_j + t_c \cdot std,error\left(\hat{\beta}\right)\right) = 1-\alpha \; .$$

This gives the 1-α confidence interval. The confidence interval is known as an interval estimate in contrast to the point estimates.

Example – Homework #4

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	93.05434	93.05434	1032.473	3.27436E-38
Residual	57	5.137275	0.090128		
Total	58	98.19162			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	4.820175	0.079173	60.8815	1.46E-53	4.661633978	4.978717	4.661634	4.978717
X Variable 1	0.073747	0.002295	32.13212	3.27E-38	0.06915094	0.078343	0.069151	0.078343

df = 57, using α = 0.05, then t_c = 2.00

Confidence interval for intercept

$$\Pr\left(4.820 - 2.00 \cdot 0.079 \le \beta_1 \le 4.820 + 2.00 \cdot 0.079\right) = 1 - .05$$
$$\Pr(4.662 \le \beta_1 \le 4.978) = 95\%$$

There is a 95% chance that the true parameter, β_1, lies between 4.662 and 4.978

$$\Pr\left(0.0737 - 2.00 \cdot 0.0023 \le \beta_2 \le 0.0737 + 2.00 \cdot 0.0023\right) = 1 - .05$$
$$\Pr(0.0691 \le \beta_2 \le 0.0783) = 95\%$$

There is a 95% chance that the true parameter, β_2, lies between 0.0691 and 0.0783.

If a confidence interval includes zero, then you cannot rule out that the parameter equals zero. The estimate for the parameter would have an insignificant t-statistic.

Prediction Confidence Intervals

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