Lecture Notes 5

1. Goodness-of-Fit

The goodness-of-fit measure is, R^2.

$$R^2 = 1 - \frac{SSE}{SST}$$

If $R^2 = 0$, then no fit
If $R^2 = 1$, then a perfect linear fit
Also, n = k which is algebraic system

Problem – As the number of x variables increases, R^2 always gets larger

Adjusted R^2

Penalize the goodness of fit if more variables are added

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-k}\ .$$
$$\bar{R}^2 = 1 - error \cdot penalty$$

As the number of independent variables increase, the penalty increases, but the error could decrease if new variables explain ‘y’ better.

Sometimes $\bar{R}^2$ can be negative, indicating a very poor fit

Note – Very important; it has to be the same y variable
One model it cannot be y and in another it is ln (y)

2. Maximum Likelihood

Assume the residuals and y variable has a known distribution, usually the normal distribution.
An algorithm finds the parameters that maximizes this distribution
The total value of the distribution is L(…) or ln L(….)

Distributions are not linear, like the normal
Algorithm is nonlinear and uses an iteration technique to find parameters that max. distribution

Equation for normal distribution is below:

$$f(x_i) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{ -\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2 \right\}$$

If you have a regression equation, the parameter estimator is,
$\hat{\beta} = \left(x^T x\right)^{-1} x^T y$

However, the estimate for the variance is biased.

Many techniques in times series assumes a normal distribution.

Two more goodness of fit measures are based on maximum likelihood.

Metrics

Information criterion = penalty + reward

Penalty – increases as more parameters are added to model
Reward – more parameters mean a better fit, so reward becomes smaller

Choose the model that gives the small information criterion.

Akaike's information criterion (AIC) –

$$AIC = penalty + reward = 2k + n\left[\ln\left(\frac{SSE}{n}\right)\right]$$

Note – remember: the number of observations will stay the same
Choose the model that gives AIC

Schwarz Information Criterion (SIC) (or Bayesian Information Criterion)

$$SIC = penalty + reward = k\ln(n) + n\left[\ln\left(\frac{SSE}{n}\right)\right]$$

Note – the reward term is the same for the AIC and SIC

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One model it cannot be y and in another it is ln (y)